III. Low Noise Amplifiers
Outline

- Figures of merit
- Basic Structure
- Input and output matching
- Noise (NF)
- Gain (over the band)
- Linearity (IIP3, SFDR)
- Stability
Figures of Merit (FOM)

- Frequency
- Noise Figure
- Linearity (CP-1dB, IIP3)
- Bandwidth and Q
- Gain S21
- Power Consumption
- Supply Voltage
- Stability
## Typical Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>NF</td>
<td>~ 2dB</td>
</tr>
<tr>
<td>IIP3</td>
<td>~ 0 dBm</td>
</tr>
<tr>
<td>CP-1dB</td>
<td>~ -10 dBm</td>
</tr>
<tr>
<td>Gain</td>
<td>15~ 20 dB</td>
</tr>
<tr>
<td>S11, S22</td>
<td>&lt; -10 dB</td>
</tr>
<tr>
<td>S12</td>
<td>&lt; - 30 dB</td>
</tr>
<tr>
<td>Supply Voltage</td>
<td>~ 1.2-1.8 V</td>
</tr>
<tr>
<td>Current</td>
<td>~ 5 mA</td>
</tr>
</tbody>
</table>
CS Structure
Dominant Pole

The input cap is usually the dominant pole:

\[ \omega_0^{-1} \approx R_s (C_{in} + |A_v| C_u) \]

\[ \omega_0^{-1} \approx R_s C_{in} (1 + |A_v| u) \approx R_s C_{in} |A_v| u \]
Dominant Pole

For a voltage gain of $g_m R_L$, at low freq., we have:

$$\omega_0^{-1} \approx R_s C_{in} |A_V| u = (g_m R_s) (g_m R_L) \frac{C_{in}}{g_m} u$$

$$= |A_v|^2 \frac{R_s}{R_L} \frac{1}{\omega_T} u$$

$$\omega_0 |A_v|^2 = \frac{R_L}{R_s} \frac{1}{\omega_T} u$$

If $A_v=10$ and $R_L/R_s=2$, $u=0.2$

$$\omega_0 = \frac{\omega_T}{10}$$
**Observations**

- For high frequency operation
  - Higher $\omega_T$ is desirable $\Rightarrow$ high $g_m$ $\Rightarrow$ high current
  - Less capacitance is desirable
  - Lower gain $\Rightarrow$ reduce Miller capacitance? …but we usually need a certain amount of gain. So what to do? (Think….cascode?)
Input Matching--Why

• Provides Good Termination for Transmission Line from Antenna (Antenna/RF filters usually are designed with 50 Ω terminations, same as the measurement equipments)
• Maximizes Power Transferred
  • Conjugate matching
• Preserves Characteristics of Duplexer Filter
• Stability

Connection length \( \geq \lambda/4 \) should be considered a transmission line
Extra Noise from $R_T$ => $NF \geq 3 \text{ dB}$
Input Noise Source is Attenuated => NF is degraded further

![Diagram]

$$Z_{in} = R_T = 50 \ \Omega$$
Input Matching - 1/Gm Termination

Require Large Power to Achieve 50-Ohm Matching
Unity Current Gain => Current Noise becomes Significant and Limited Reverse Isolation

\[ Z_{\text{in}} = \frac{1}{g_m} = 50 \, \Omega \]
Input Matching - 1/Gm Termination

\[ F = 1 + \frac{v^2_{eq}}{4kTR_s} \]

BJT: \( F_{\text{min}} = 1 + \frac{1}{2g_m R_s} = 1.5 = 1.8 \text{dB} \)

MOS: \( F_{\text{min}} = 1 + \frac{2}{3g_m R_s} = 1.7 = 2.2 \text{dB} \)
Input Matching--Resistive Shunt Feedback

- Added noise
- Broadband Amplification due to Feedback => More Power to Reduce Noise

\[ Z_{\text{in}} = \frac{R_F}{1+Av} = 50 \, \Omega \]
• Ideal Inductors are Noiseless $\Rightarrow$ No Extra Noise Contribution
• More Flexibility in Design and Optimization

$Z_{in} = 50 \ \Omega$
Input Matching--Inductive Degeneration

\[ Z_{in} = Z_{L_G} + Z_{C_{gs}} + (1 + g_m Z_{C_{gs}}) Z_{L_S} \]

\[ = s(L_G + L_S) + \frac{1}{s C_{gs}} + \frac{g_m L_S}{C_{gs}} \]

\[ = \omega_T L_S \quad \text{at} \quad f = \frac{1}{2\pi \sqrt{(L_G + L_S) C_{gs}}} \]
Input Matching--Inductive Degeneration

• Low-Q Inductors:
  • Relatively Broad Band
  • Small Design Headroom for Matching
• Small Noise => Large Gm => Small L_S => Large L_G => L_G parasitic resistance contributes more noise
• Inductors, in particular L_G, have been implemented using bondwire or off-chip
• $NF$ and gain of the LNA determine $NF_{total}$
• LNA is there to improve the overall noise performance ($NF_{Mix} > NF_{LNA}$)
• High $IIP3$ of LNA is usually desired for reducing in-band IM distortions
• Total $IIP3$ is usually determined by Mixer and the following blocks, not by LNA
• Matching is optimized for overall signal performance in the front-end
LNA - Noise Figure

\[ i_{n,d}^2 = 4KTg_d0 \Delta f \]

\[ i_g^2 = 4KT \delta g_g \]

\[ g_g = \frac{\omega^2 C_{gs}^2}{5g_{d0}} \]

\[ C = \frac{i_g i_d^*}{\sqrt{i_g^2 i_d^2}} \]
LNA - Noise Figure

\[
NF = 1 + \frac{i_u^2 + |Y_C + Y_s|^2 e_n^2}{i_s^2}
\]

\[
NF = 1 + 2R_n (G_{opt} + G_C) + \left(\frac{R_n}{G_s}\right) |Y_S - Y_{opt}|^2
\]

\[
Y_{opt} = G_{opt} + jB_{opt} = \sqrt{G_u / R_n + G_c^2} - jB_c
\]

\[
e_n^2 = R_n 4KT\Delta f
\]

\[
i_s^2 = G_S 4KT\Delta f
\]

\[
i_u^2 = G_u 4KT\Delta f
\]

\[
i_c^2 = Y_C e_n^2
\]

\[
Y_S = G_S + jB_S
\]

\[
Y_C = G_C + jB_C
\]
The current gain of the MOS Amplifier is given by:

\[ i_o = g_m v_1 = g_m \left( \frac{v_s}{R_s + R_g + \frac{1}{j\omega C_{gs}}} \right) \]

\[ = g_m \left( \frac{v_s}{(R_s + R_g) j\omega C_{gs}} + 1 \right) \approx g_m \left( \frac{v_s}{(R_s + R_g) j\omega C_{gs}} \right) \]
LNA - Noise Figure

• This can be re-written as:

\[ i_o = G_m v_s \]

\[ G_m = -j \omega_T \frac{1}{\omega (R_s + R_g)} \]

• The total noise is given by:

\[ \bar{i}_{o,T}^2 = G_m^2 (\bar{v}_g^2 + \bar{v}_s^2) + \bar{i}_d^2 \]

• The noise figure is given by:

\[ F = 1 + \frac{\bar{v}_g^2}{\bar{v}_s^2} + \frac{\bar{i}_d^2}{G_m^2 \bar{v}_s^2} \]
LNA - Noise Figure

• Substitution of the various noise sources leads to:

\[ F = 1 + \frac{R_g}{R_s} + \frac{\lambda}{\alpha} \frac{g_m}{R_s} \left( \frac{\omega}{\omega_T} \right)^2 (R_s + R_g)^2 \]

• Assume \( R_s >> R_g \)

\[ F = 1 + \frac{R_g}{R_s} + \left( \frac{\omega}{\omega_T} \right)^2 \left( \frac{\lambda}{\alpha} g_m R_s \right) \]

It’s important to note that this expression contains both the channel noise and the gate induced noise. If we assume that \( R_g = R_{\text{poly}} + 1/5 g_m \), and the noise is independent from the drain thermal noise, we get a very good approximation to the actual noise without using correlated noise sources.
The optimal value of $R_s$

\[
\frac{\partial F}{\partial R_s} = -\frac{R_g}{R_s^2} + \left(\frac{\lambda}{\alpha}\right)\left(\frac{\omega}{\omega_T}\right)^2 g_m = 0
\]

\[
R_g = \left(\frac{\lambda}{\alpha}\right)\left(\frac{\omega}{\omega_T}\right)^2 g_m
\]

\[
R_{s,\text{opt}} = \sqrt{R_g\left(\frac{\lambda}{\alpha}\right)\left(\frac{\omega}{\omega_T}\right)^2 g_m} = \left(\frac{\omega_T}{\omega}\right)^2 \sqrt{\frac{R_g}{\left(\frac{\lambda}{\alpha}\right)g_m}}
\]

\[
F_{\text{min}} = 1 + 2\left(\frac{\omega}{\omega_T}\right)\sqrt{g_m R_g \left(\frac{\lambda}{\alpha}\right)}
\]
LNA - Noise Figure

Let’s find \( R_{s,\text{opt}} \) for a typical amplifier. Say \( f_T = 75\text{GHz}, f = 5 \text{ GHz}, \) and \( (\gamma/\alpha) = 2. \) Also suppose that by proper layout \( R_{\text{poly}} \) is very small. The intrinsic gate resistance is given by:

\[
R_g = R_{\text{poly}} + \frac{1}{5g_m} \cong \frac{1}{5g_m}
\]

\[
\frac{R_g}{R_s} = 0.1 \quad \frac{1}{5g_m R_s} = 0.1 \quad \Rightarrow \quad g_m = \frac{10}{5 \times 50\Omega} = \frac{1}{25} S = 40\text{ms}
\]

Note that for \( V_{gs} - V_T = 200\text{mV}, \) the required current is:

\[
I_{ds} = \frac{1}{2} g_m (V_{gs} - V_T) = \frac{1}{2} 40\text{ms} \times 200\text{mV} = 4 mA
\]

\[
R_{s,\text{opt}} = \frac{f_T}{f} \sqrt{\frac{R_g}{(\frac{\lambda}{\alpha})g_m}} = 15 \sqrt{\frac{5 \cdot 25}{2}} \cong 119\Omega
\]

\[
F_{\text{min}} = 1 + 2 (\frac{\omega}{\omega_T}) \sqrt{g_m R_g (\frac{\lambda}{\alpha})} = 1 + \frac{2}{15} \sqrt{\frac{5 \times 2}{25}} = 1.08 = 0.35\text{dB}
\]
LNA - Noise Figure

• for a given circuit / bias source admittance $Y_s$;
• there is an optimum admittance to $F_{\text{min}}$!
• the RF Filter / Duplexer impedance $R_s = 50 \, \Omega$ ($Y_s =$ matching is a must!)
• the NF optimization is fixed), but by changing the bias and W/L ratios circuit
• it is difficult/impossible $50 \, \Omega$ input matching and $F_{\text{min}}$
• The noise is higher at higher frequencies
LNA - Noise Figure

\[ Z_{in} \]

\[ R_L \]

\[ L \]

\[ R \]

\[ Z_{in} \]

\[ R_L \]

\[ L \]

\[ R \]
It’s fairly easy to calculate the noise for the case with inductive degeneration. Simply observe that the input generators noises see a gain of $G_m^2$ to the output. The drain noise, though, requires a careful analysis. Since it flows partly into the source of the device, it activates the $g_m$ of the transistor which produces a correlated noise in shunt with drain noise.
The above equivalent circuit shows that the noise component flowing into the source is given by the current divider:

\[ v_\pi = \left(-g_m v_\pi + i_d\right) \times \frac{j \omega L_s}{j \omega L_s + \frac{1}{j \omega C_{gs}} + j \omega L_g + R_s} \times \frac{1}{j \omega C_{gs}} \]
At resonance:

\[ v_\pi = -(g_m v_\pi + i_d) \times \frac{j\omega L_s}{R_s} \times \frac{1}{j\omega C_{gs}} \]

\[ v_\pi = -(g_m v_\pi + i_d) \times \frac{L_s}{R_s C_{gs}} \]

\[ v_\pi (1 + \frac{g_m L_s}{R_s C_{gs}}) = -i_d \frac{L_s}{R_s C_{gs}} \]

When matched, \( R_s = \omega T L_s \)

\[ 2v_\pi = -i_d \frac{L_s}{R_s C_{gs}} \quad g_m v_\pi = -i_d \frac{g_m L_s}{2 R_s C_{gs}} = -\frac{i_d}{2} \]
So we see that only 1/4 of the drain noise flows into the output! The total output noise is therefore

\[
i_{o,T}^2 = G_m^2 (\overline{v_g^2} + \overline{v_s^2}) + \frac{i_d^2}{4}
\]

\[
F = 1 + \frac{v_g^2}{v_s^2} + \frac{i_d^2}{4v_s^2 G_m^2}
\]

\[
F = 1 + \frac{R_g}{R_s} + \left(\frac{\omega}{\omega_T}\right)^2 \left(\frac{\lambda}{\alpha}\right) g_m \left(2R_s\right)^2 4R_s
\]

\[
F = 1 + \frac{R_g}{R_s} + \left(\frac{\omega}{\omega_T}\right)^2 \left(\frac{\lambda}{\alpha}\right) g_m R_s
\]

The inductive degeneration did not raise the noise! So the minimum noise figure \(F_{\text{min}}\) is the same.

The advantage is that the input impedance is now real and programmable \((\omega T L_s)\). By proper sizing, it's possible to obtain a noise and power match.
Either By Short-Cut or By Series Resonant Tank:

\[ G_m = g_m \frac{Z_{C}}{R_s + Z_{in}} \]

\[ (= g_m Q_{in}) \]

\[ \therefore G_m \bigg|_{\omega = \omega_0} = g_m \frac{Z_{C}}{2 R_s} \frac{g_{m} Z_{C} g_{s}}{\omega_T} = \frac{\omega_T}{2 \omega_0 R_s} \]
LNA - Effective Gm

\[ W' = \alpha W, \quad L' = L, \quad V'_{GS} = V_{GS}, \quad L'_G = L_G / \alpha \]

\[ \therefore I'_D = \alpha I_D \]

\[ \therefore \omega'_T = \omega_T \]

\[ \therefore G'_m = \frac{\omega'_T}{2\omega'_o R_s} = \frac{\omega_T}{2\omega_o R_s} = G_m \]

\[ \therefore \text{Same } G_m \text{ as } C_{gs} \text{ and Power are Reduced!} \]
LNA – Linearity Consideration

\[ i_d = g_m V_{gs} + b V_{gs}^2 + c V_{gs}^3 \]

\[ i_{in} = \frac{V_{in}}{2R_S} \]

\[ |V_{gs}| = \frac{V_{in}}{\omega_0 C_{gs}} = \frac{V_{in}}{2R_S \omega_0 C_{gs}} = \frac{V_{in} Q}{2} \]

\[ i_d = g_m \frac{V_{in} Q}{2} + b \left( \frac{V_{in} Q}{2} \right)^2 + c \left( \frac{V_{in} Q}{2} \right)^3 \]

\[ A_{in}^2 \text{IP3} = \frac{4 g_m Q/2}{3c(Q/2)^3} = \frac{16g_m}{3cQ^2} \]

Larger \( g_m \) or Smaller \( Q \) \( \rightarrow \) larger IP3 \( \rightarrow \)
larger power consumption or larger \( C_{gs} \)
LNA – Linearity Consideration

- Maximize Q or $V_{gs}$ => NF Degradation
  - For Same Power, Minimize Device Size
  - For Same Device Size, Maximize Power
- Maximize Supply Voltage
- Minimize Gain
- Employ Differential Topology
LNA – Output Loading

- Small Resistive Load $\Rightarrow$ Small Gain
- Large Resistive Load $\Rightarrow$ Problems with Bandwidth, Voltage Headroom and Linearity
- Inductors Resonate Output Capacitance for High Frequency, High Gain, and Narrow-Band Filtering
LNA – Output Loading

- For Low-Q Inductors, May Need Negative-Gm Compensation to Achieve High and Tunable Q and Gain
- Negative-$G_m$ Compensation Circuitry Degrades $NF$, Linearity, and Power
LNA – Output Matching Network

- Driving Off-Chip Filter Needs Output Matching:
  - Preserve Filter Characteristic
  - Need 50-Ohm Buffer
  - Consumes Power and Degrades Linearity
- LC Resonant Loading Can Also Be Used for Matching
- Driving On-Chip May NOT Need Matching
LNA – Stability Criteria

\[ S_{11} = \frac{Z_{in} - R_s}{Z_{in} + R_s}, \quad S_{22} = \frac{Z_o - R_L}{Z_o + R_L} \]

\[ S_{21} = A_P = \frac{P_o}{P_{in}}, \quad S_{12} = \frac{P_{in}}{P_o} \]

\[ \Delta = S_{11} S_{22} - S_{21} S_{12} < 1 \]

\[ K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2 |S_{21}| |S_{12}|} > 1 \]
LNA – Cascode Design

- Increase Reverse Isolation => Improved Stability
- Increase Voltage Gain
- Eliminate Input Miller Capacitance
- Sacrifice Voltage Headroom and Linearity
### Comparison

<table>
<thead>
<tr>
<th></th>
<th>(Z_{in})</th>
<th>F(50 Ohm)</th>
<th>Gain</th>
<th>Typical NF (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CS/Shunt Resistor</td>
<td>(R_{sh})</td>
<td>(2 + \frac{4\gamma}{g_m R_s})</td>
<td>(-\frac{g_m R_L}{2})</td>
<td>10</td>
</tr>
<tr>
<td>Common Gate</td>
<td>(1/g_m)</td>
<td>(1 + \gamma)</td>
<td>(g_m R_L)</td>
<td>5</td>
</tr>
<tr>
<td>Feedback Stage</td>
<td>(\frac{R_F + R_L}{1 + g_m R_L})</td>
<td>(1 + \frac{\gamma}{g_m R_s} + \frac{R_s}{R_F} (1 + \frac{1}{g_m R_s})^2) (-g_m R_L)</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>Inductor Source Degeneration</td>
<td>(\frac{g_m L_s}{C_{gs}})</td>
<td>(1 + \frac{\gamma}{g_m R_s} \left(\frac{1}{Q_{in}}\right)^2)</td>
<td>(-g_m Q_{in} R_L)</td>
<td>2</td>
</tr>
</tbody>
</table>
LNA – Differential Design

- Insensitive to Parasitic Inductors
- Reject Common-Mode Noise (Substrate, Supply)
- Improved Linearity and Dynamic Range
- Higher NF for Same Power or Same NF for Doubled Power
- Larger Chip Area

- Antenna and Duplexer Filter Are Single-Ended
- Need Baluns to Convert Single-Ended Signals to Differential
Cascode With Inter-Stage Matching

Without $L_{\text{int}}$, $M_1$ is loaded by

$$Z = \frac{1}{g_{m_2} + j \omega C_{gs2}}$$

$L_{\text{int}}$ provide inter-stage matching

- $C_a$ is for stable reason
- The gain of first stage is improved
- The NF reduced
- Larger silicon area
No stacked transistors.
$V_B$ is used to control the gain.
Which will not affect input matching.
Low Noise....
High Linearity....
hey.....are you
LLLLL....listening!